JOURNAL OF PROPULSION AND POWER
Vol. 9, No. 4, July–Aug. 1993

Verification of the Theoretical Discharge Coefficient of a Subcritical Airflow Meter

D. J. Lahti*
GE Aircraft Engines, Cincinnati, Ohio 45215
and
A. Hamed†
University of Cincinnati, Cincinnati, Ohio 45221

Since many modern high bypass ratio turbofan engines have flow rates that exceed the capacity of most of the worlds airflow calibration facilities, their airflow metering bellmouth inlets cannot be calibrated, but rather their discharge coefficients must be determined theoretically. The objective of this program was to verify the theoretically derived discharged coefficient for a scale model of such a bellmouth which was small enough to allow its calibration in an existing laboratory facility. Extensive flowfield measurements were also made to further validate theoretical predictions. Thus, this program provides a "calibration" of the theoretical method used, and establishes a link between a traceable airflow metering standard and large engine bellmouth inlets that cannot be calibrated any other way.

Introduction

THE need for highly accurate airflow measurement in aircraft engine ground testing is self-evident since engine thrust is directly proportional to airflow rate. Historically, when high-accuracy compressible airflow measurements are required, flow meters designed for critical (choked flow) conditions are used.[1] Over the period of the past 30 yr, two standard critical flow meter designs have received wide acceptance throughout industry.[2] These standard flow meters are the Smith and Matz[3] critical flow circular-arc throat venturi and the ASME critical flow nozzle. Both Smith and Matz[1,3] and Stratford[4] discuss many of the advantages of metering airflow using critical flow venturis.

Although it is desirable to utilize critical flow meters whenever possible, because of their inherent accuracy "advantage," it is not possible in the case of an aircraft engine bellmouth. The supersonic diffuser shock wave and attendant distortion downstream of its throat could not be tolerated by the engine fan or compressor. Therefore, engine bellmouths require subcritical flow conditions. In addition, modern flow metering bellmouths are designed to simulate as nearly as possible the flow conditions that would exist at the engine face when it is operating with its real flight inlet. This includes the radial velocity profile at the fan or compressor face as well as the tip boundary layer. This is done to help assure the ground test-derived engine performance will be as nearly representative as possible of the engine's performance when in flight.

Because of this, today's engine inlet bellmouths have geometries that are not typical of industry standard flow meters for which a large body of discharge coefficient calibration data exists. Finally, some of today's engines have flow rates as high as 2500 lb/s, and some engines planned for the near future will have flow rates exceeding 3000 lb/s. Thus, the possibility of calibrating them in any of the world's largest engine test facilities, such as the Arnold Engineering Development Center (AEDC) in Tullahoma, Tennessee is now remote, and will soon be impossible, since the highest known flow rate of any of these facilities is about 2750 lb/s.[5]

The only known way to determine the discharge coefficients of bellmouths having subcritical flow conditions and nonstandard geometries which cannot be calibrated elsewhere, is to determine them theoretically.

There are two ways of validating the theoretical method used for this purpose. The first is to utilize the method used for the large bellmouth discharge coefficient predictions and apply it to a scale model bellmouth of small enough size such that it can be calibrated using an industry standard critical flow venturi. The second is to directly compare the detailed flowfields measured in such a scale model bellmouth with those predicted. This approach is similar to that used by Smith and Matz[6] in their attempt to better understand the validity of the discharge coefficient of an ASME nozzle obtained from incompressible (water) calibration for use as a compressible subcritical airflow meter. They did not show comparisons of predicted vs measured discharge coefficients, but they did show comparisons of their flowfield predictions with measurements taken over a range of subcritical throat Mach numbers. Although their predictions of the ASME nozzle flowfield compared reasonably well with their measurements, since predicted discharge coefficients were not presented and the throat boundary layers were not measured in their tests, it is difficult to draw conclusions from their work alone as to the adequacy of theoretical methods for the prediction of the discharge coefficients of subcritical nonstandard (i.e., large engine bellmouth) flow meters.

It should be noted that the work of Smith and Matz[6] was accomplished nearly 30 yr ago, and to the author's knowledge, there has been virtually no published work since then which attempted to build on their experience. The partial success of Smith and Matz[6] in predicting subcritical ASME nozzle flowfields, and the myriad successes in the use of CFD since then, along with the continued push for increased flow metering accuracy in large engine bellmouths whose discharge coefficients could not be determined experimentally, led to the test program and theoretical studies which are described below. The main objective of this program was to experimentally verify the theoretically predicted discharge coefficient of a scale model of a large engine bellmouth using an industry standard ASME critical flow metering nozzle. The rationale for this was that once the prediction method had

Received Dec. 30, 1992; revision received March 5, 1993; accepted for publication March 9, 1993.

*Manager, Aerothermo Systems Integration. Senior Member AIAA.
†Professor, Department of Aerospace and Engineering and Applied Mechanics. Fellow AIAA.

been verified with an accepted industry standard flow meter, the method could be used on any subcritical flow meter, regardless of size, and in particular, very large engine bellmouths where other means are not available for establishing their discharge coefficients. It was also intended that the data obtained could be used by others to serve as a bench mark for establishing the accuracy of other theoretical methods used for the same purpose.

Background

A head-type flow meter is one that uses the measured difference between the total pressure and static pressure in the meter throat to compute the so-called velocity head (hence, the name "head meter") which is subsequently used to compute the flow rate. When the static pressure (and thus velocity) varies across the throat of the meter, this effect must be taken into account in order to calculate the flow rate. The discharge coefficient of a head-type flow meter is defined as the ratio of the actual flow rate to the ideal one-dimensional inviscid flow rate that would exist if all the flow were at the velocity corresponding to the static pressure at the meter throat wall. The more uniform the velocity profile across the throat of the meter, the higher its discharge coefficient. A discharge coefficient of unity would imply a perfectly uniform velocity profile and no wall boundary layer. Therefore, the accurate determination of the discharge coefficient of a subcritical flow meter is dependent on three primary factors. The first is the velocity level that exists at the throat, the second is the profile of velocity (pressure) across the throat, and the third is the thickness of the throat wall boundary layer.

There are four methods for determining the discharge coefficient of a head-type flow meter. The first is to survey the velocity, pressure, and temperature fields across the meter throat to determine the ρV profile which can then be integrated to determine the actual mass flow rate. The second is to compute these same quantities using a theoretical method

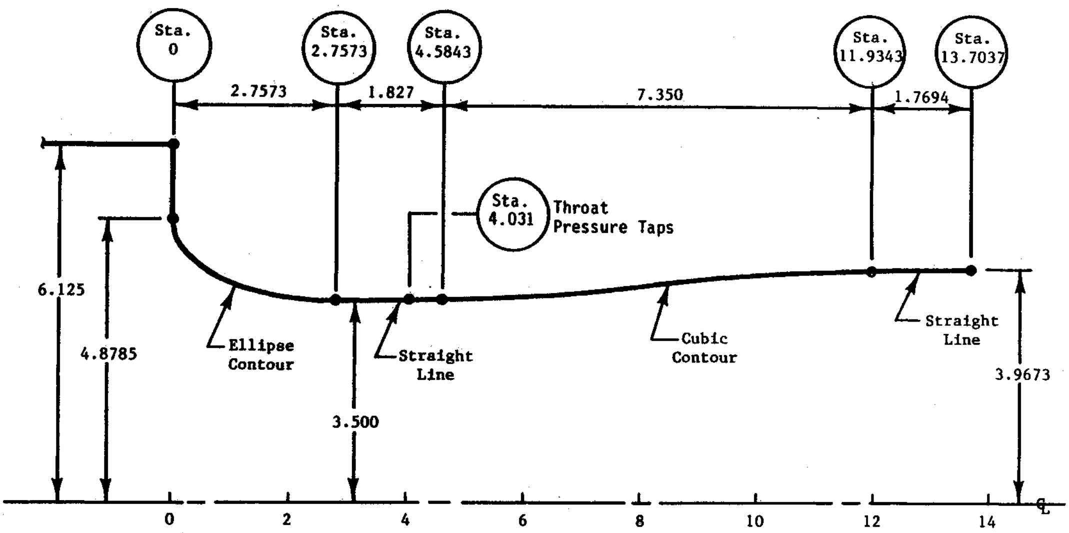

Fig. 1 Flow meter aerodynamic contour definition.

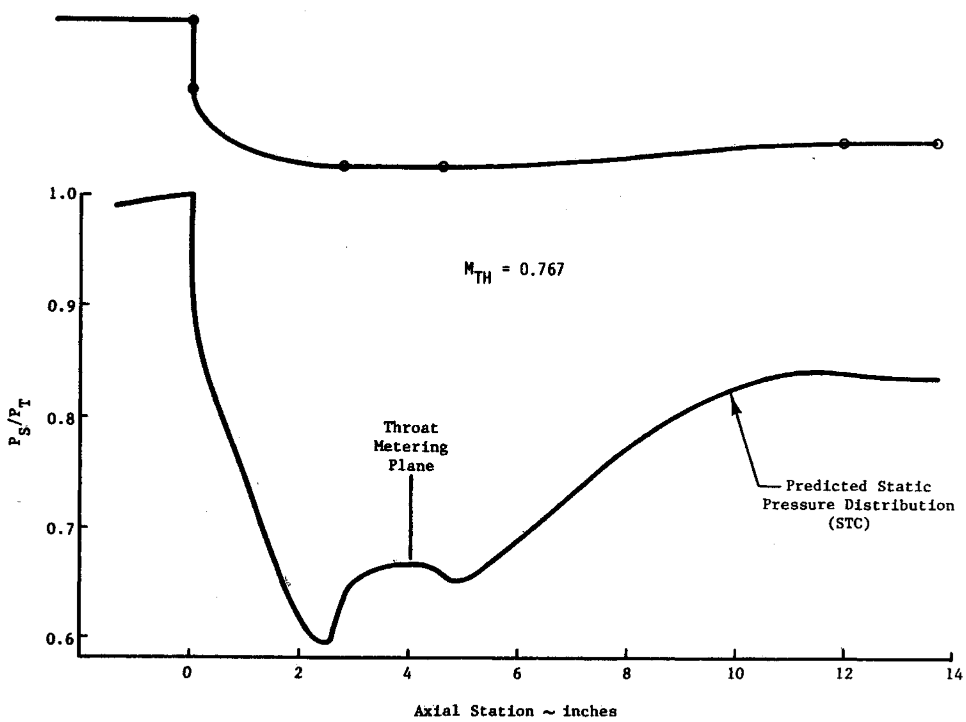

Fig. 2 Flow meter theoretical pressure distribution.

and then perform the same integration of the ρV profile. The third is to physically weigh the quantity of fluid that is passed through the meter in a given amount of time. This is called a primary calibration because the actual mass is measured directly. The fourth is to calibrate the flow meter using another flow meter whose discharge coefficient is already known. This type of calibration is called a secondary method, because the actual mass flow through the meter is not measured directly, but rather, it is calculated using the known discharge coefficient of another flow meter.

Many of the issues relating to the accuracy of determining discharge coefficient using these four methods are discussed in detail in Ref. 2. The test program described below was designed to allow the determination of the discharge coefficient of the simulated bellmouth inlet using two of the four methods mentioned above. The first was to make flow surveys across the throat of the meter including the wall boundary layer. The second was to perform a secondary calibration of the simulated bellmouth using an industry standard ASME critical flow venturi. The results of each of these tests and comparisons with the theoretically determined discharge coefficients are now described.

Meter Design and Pretest Predictions

Flow Meter Design

The flow meter aerodynamic contour definition is shown in Fig. 1. Figure 2 shows the aerodynamic contour along with the theoretically predicted wall pressure distribution. In Fig. 1, it is seen that the meter consists of four main segments. The contracting section has an elliptical contour from the so-called "hilite" station ($X = 0$, $R = 4.8785$). The upstream 6.125-in.-radius pipe is part of the facility adapter hardware. A radial line connects the two at station 0. The throat region of the meter consists of a 1.827-in.-long cylindrical section from the end of the elliptical section to the start of the diffuser.

The cylindrical section which connects the upstream contraction with the downstream diffuser is intended to provide a region of constant physical flow area for ease of area measurement. Since the absolute flow rate depends on the actual throat area, a cylindrical section is chosen to minimize the area measurement uncertainties. Although this may not be a critical problem for small meters whose contour can be inspected with great accuracy on modern contour measuring machines, it is a problem on very large meters whose size prohibits this possibility. Table 1 shows the theoretical coordinates compared with the inspected coordinates of the meter.

Pretest Predictions

The stream tube curvature (STC) computer program[7] was selected for use in predicting the meter flowfield and potential component of discharge coefficient. It was chosen because the author's many years of experience with it, for predicting all subsonic internal flowfields, has shown it to give excellent results. In addition, a key feature of the STC program is that it is driven by global mass conservation as part of its solution procedure, thus making it a natural choice for such predictions, as opposed to other methods where numerical dissipation can produce global mass conservation errors.

The boundary-layer calculations were carried out using the Harris code.[8] It is one of a number of validated reliable boundary-layer codes, and its selection was based primarily on its being widely used and its thorough documentation.[9]

Figure 3 shows the predicted discharge coefficient vs wall static pressure ratio. The equation given in Fig. 3 is simply derived from a curve fit of the calculated points for the natural transition case. The meter discharge coefficient depends on only two components, an inviscid part, $C_{D_{\rm pot}}$, that accounts for the radial pressure variation in the throat, and a viscous part, $C_{D\delta^*}$, that accounts for the effective boundary-layer blockage. Reference 2 describes how each of these components is determined from the STC and boundary-layer solutions.

Table 1 Bellmouth model inspected coordinates

X	$R_{\rm theo}$	$R_{\rm inspect}$	X	$R_{\rm theo}$	$R_{\rm inspect}$
0.0059	4.7887	4.7964	3.6700	3.5000	3.5001
0.0161	4.7297	4.7290	3.6700	3.5000	3.5007
0.0264	4.6883	4.6879	3.6700	3.5000	3.5000
0.0367	4.6546	4.6545	4.6190	3.5000	3.4984
0.0469	4.6254	4.6226	5.0290	3.5049	3.5035
0.0674	4.5757	4.5713	5.4391	3.5175	3.5164
0.0878	4.5333	4.5293	5.8491	3.5368	3.5367
0.1084	4.4959	4.4925	6.2592	3.5617	3.5609
0.1596	4.4163	4.4131	6.6692	3.5915	3.5914
0.2109	4.3484	4.3470	7.0793	3.6250	3.6260
0.2622	4.2919	4.2887	7.4893	3.6613	3.6620
0.3647	4.1934	4.1901	7.8994	3.6994	3.7005
0.5697	4.039	4.0362	8.3094	3.7384	3.7395
0.7747	3.9205	3.9178	8.7195	3.7773	3.7782
0.9798	3.8247	3.8225	9.1295	3.8151	3.8157
1.1848	3.7462	3.7439	9.5396	3.8508	3.8505
1.3898	3.6815	3.6799	9.9496	3.8834	3.8837
1.5949	3.6285	3.6274	10.3597	3.9121	3.9126
1.7999	3.5858	3.5846	10.7697	3.9358	3.9356
2.0049	3.5523	3.5514	11.1798	3.9535	3.9530
2.2010	3.5274	3.527	11.5898	3.9643	3.9635
2.4150	3.5107	3.5100	11.6924	3.9658	3.9650
2.6200	3.5017	3.5020	11.9343	3.9673	3.9663
2.7225	3.5001	3.5010	13.7037	3.9673	3.9674

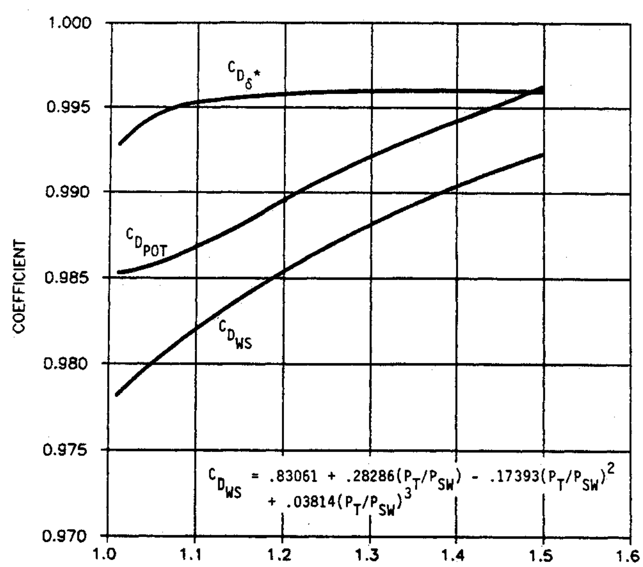

Fig. 3 Discharge coefficient components.

Test Program Description

The test facility chosen for conducting both the simulated bellmouth flowfield measurements and discharge coefficient secondary calibration, was the Medicine Lake Aerodynamics Laboratory of the FluiDyne Engineering Corporation located in Minneapolis, Minnesota. All tests were conducted in the laboratories' channel 12 cold flow static thrust stand, normally used for high-accuracy thrust and flow measurement exhaust nozzle testing.[10] Based on a statistical analysis of many previous tests, it was estimated that, for all but the lowest bellmouth pressure ratio, measured discharge coefficients in this facility would have an absolute error (facility bias + random error) no greater than ±0.25% at a 95% confidence level.

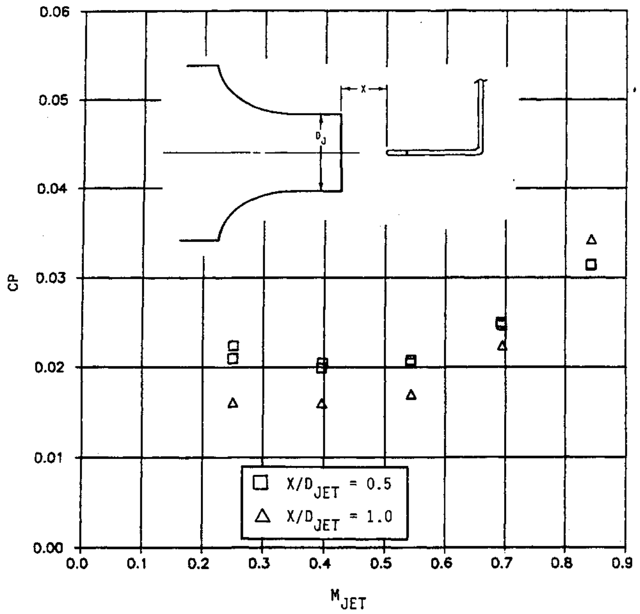

Fig. 4 Pressure survey probe free-jet calibration test results.

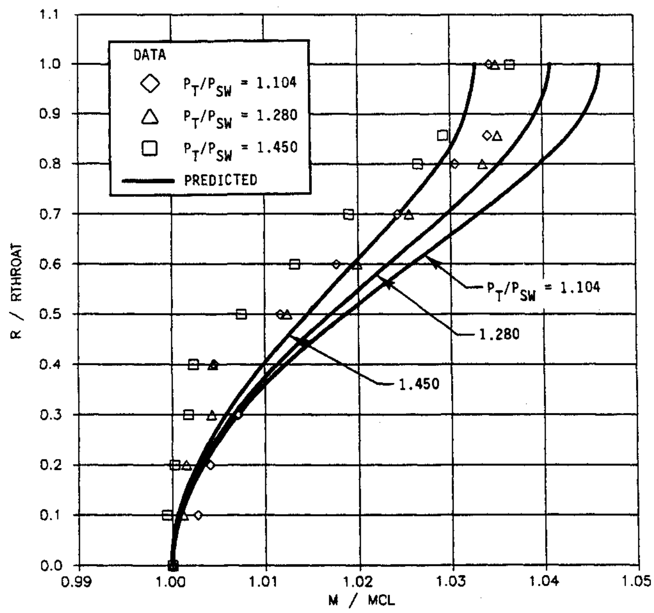

Fig. 5 Predicted and measured throat Mach profiles.

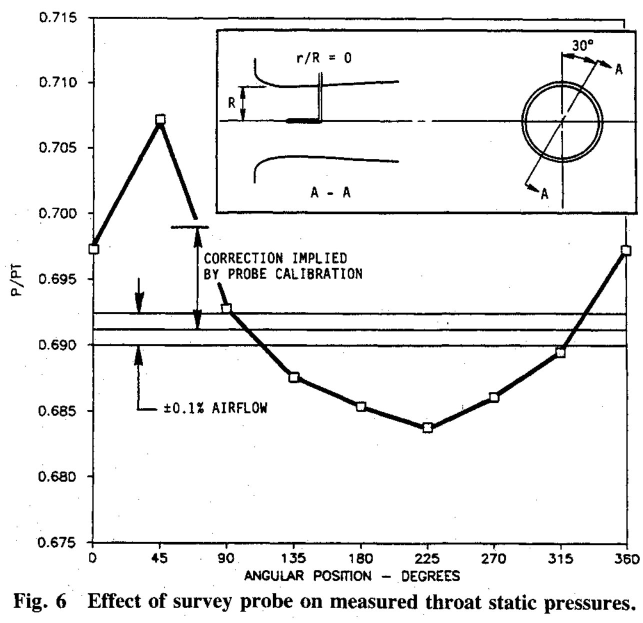

Fig. 6 Effect of survey probe on measured throat static pressures.

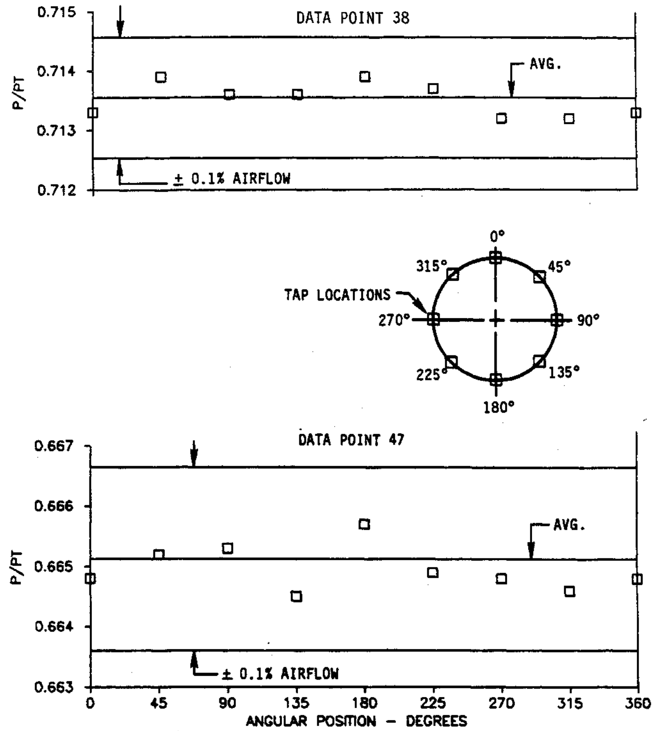

Fig. 7 Measured throat static pressures without probe.

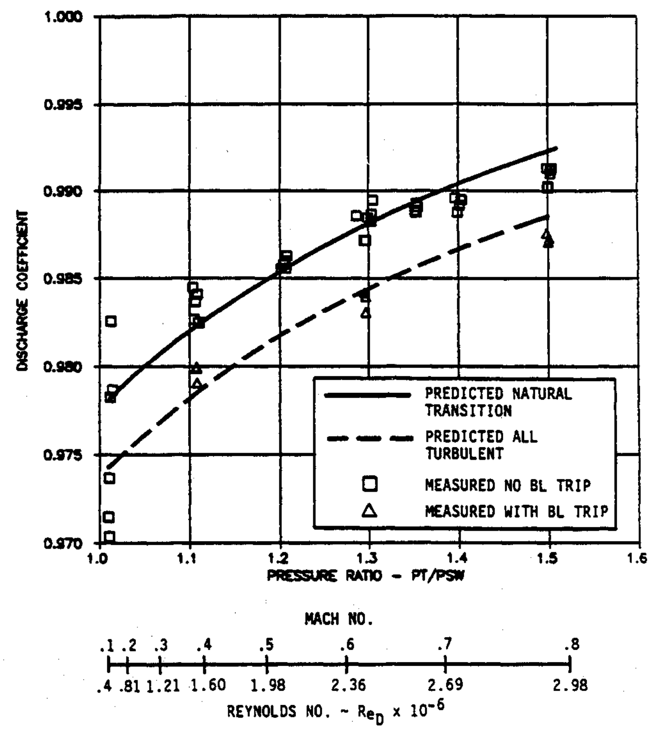

Fig. 8 Discharge coefficient vs wall static pressure ratio.

The test program consisted of surveys of the radial static pressure profiles at the meter throat, surveys of the throat wall boundary layer, and measurement of the meter overall discharge coefficient. In addition, the pitot-static probe used to conduct the throat radial static pressure profile surveys was calibrated in the free jet discharge of a 6.9915-in.-diam ASME nozzle. Details of the test facility, test apparatus, and test procedure may be found in Ref. 2.

Test Results and Discussion

The scale model test program was comprised of four phases. The first phase consisted of free-jet calibrations of the tra-

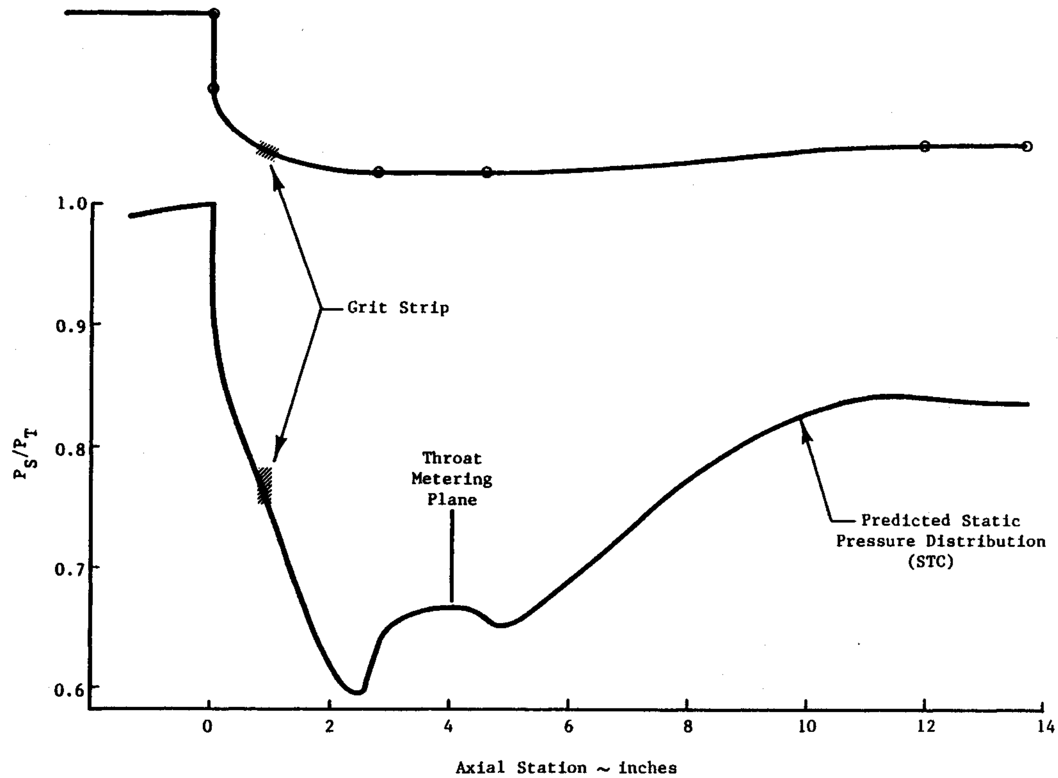

Fig. 9 Boundary-layer trip location.

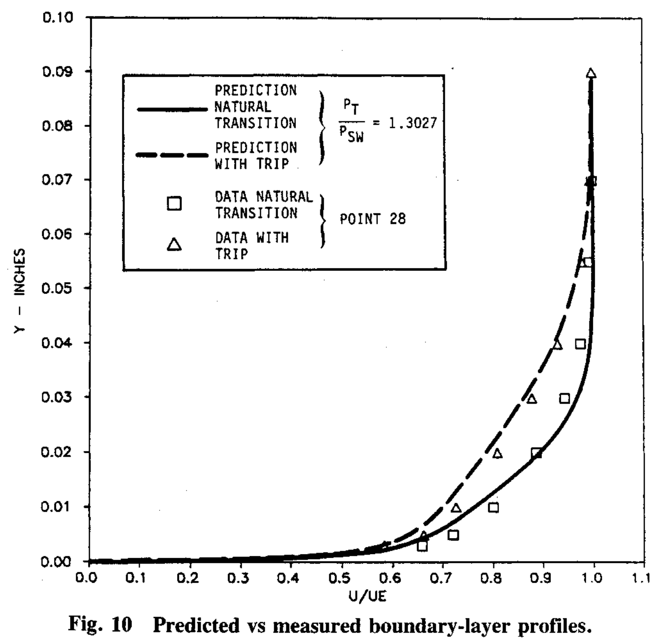

Fig. 10 Predicted vs measured boundary-layer profiles.

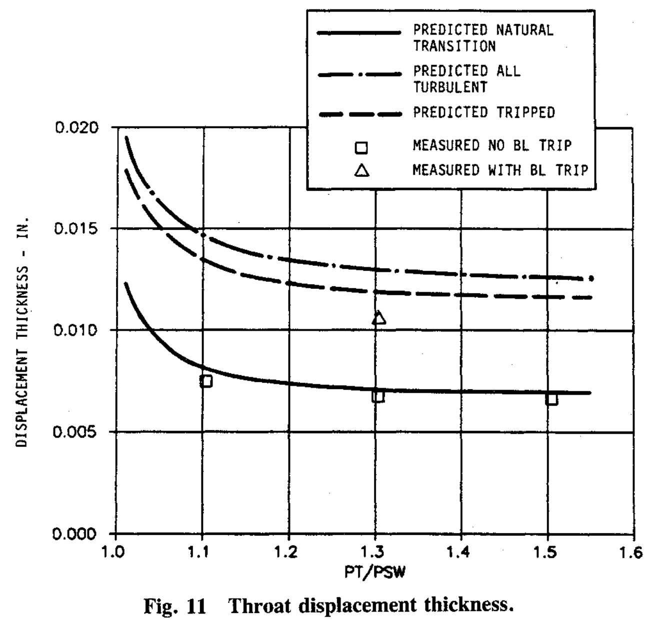

Fig. 11 Throat displacement thickness.

versing pitot-static probe used for surveying the meter throat pressure profiles. Phase 2 involved surveying the meter throat using the calibrated probe. In phase 3 the secondary calibrations of the meter discharge coefficients were obtained using the industry standard ASME low beta ratio critical flow venturi, and in phase 4 the meter throat boundary-layer surveys were acquired.

Figure 4 shows the pitot-static probe calibration results for two axial placements in the freejet. Based on Ref. 11 data, the pressure coefficient was expected to be near zero. The fact that it wasn't, illustrates why any throat survey device must be calibrated, even duplicate copies manufactured to the same specifications. In Fig. 4, the difference in CP between the two axial placements of the probe would represent a flow error of 0.1% at a jet Mach number of 0.4.

Throat static pressure profile surveys were obtained at three flow rates which spanned the operating range of the meter. Figure 5 shows the survey results adjusted for the effects of the probe calibration shown in Fig. 4. Also shown in Fig. 5 are the theoretically predicted profiles. All of the profiles shown are normalized by the centerline value of Mach number. Although it was anticipated at the outset of the program that the presence of the survey probe in the meter would have some effect on the local flowfield around the probe, it was not expected to be as large as the measurements subsequently indicated. The probe produced both a blockage effect and a flowfield distortion at the wall static pressure measuring plane. Figure 6 shows the wall static pressure distribution at the throat plane with the probe at its centerline immersion. Figure 7 shows the corresponding wall static pressure distribution

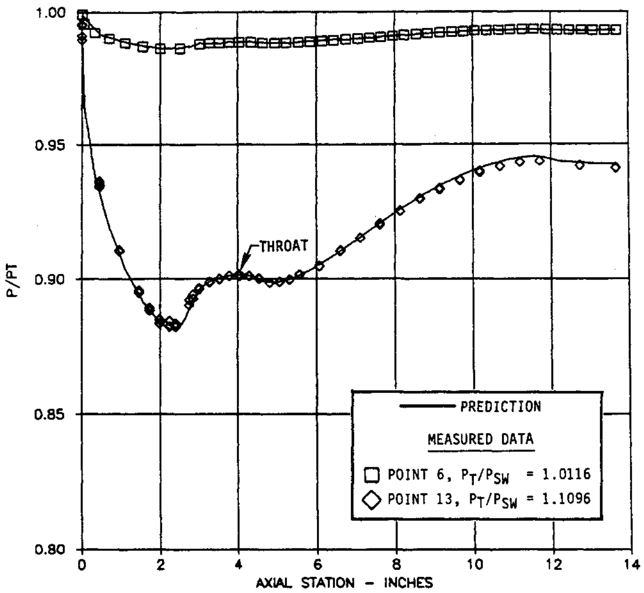

Fig. 12 **Predicted vs measured axial wall static pressure distributions.**

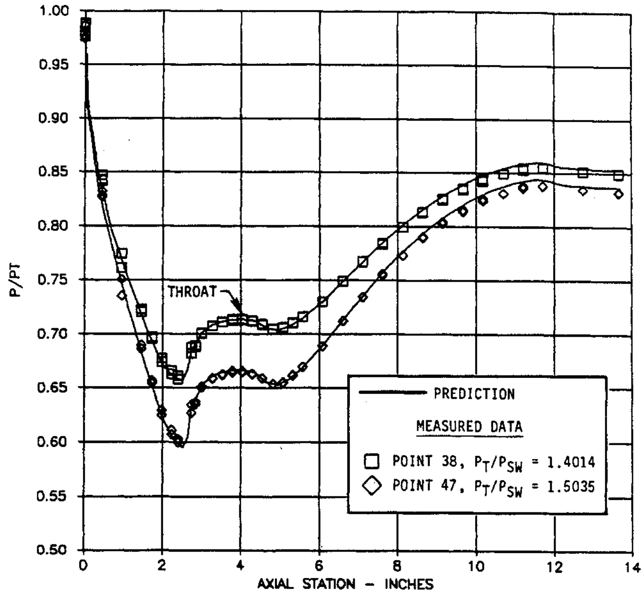

Fig. 14 **Predicted vs measured axial wall static pressure distributions.**

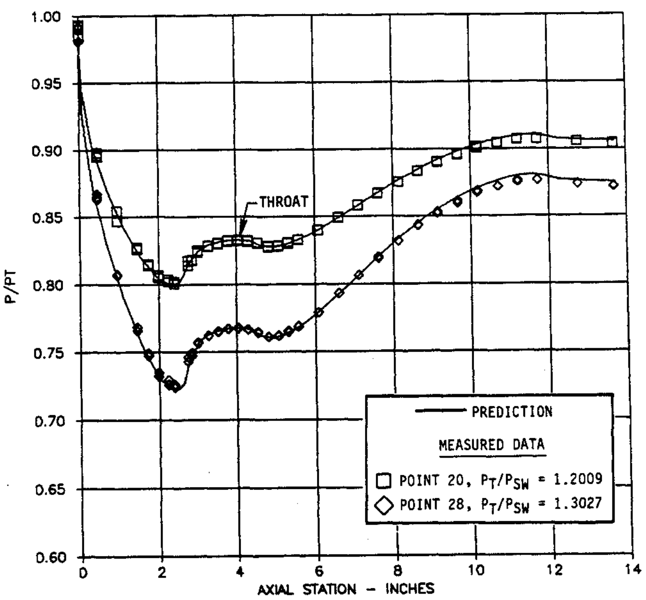

Fig. 13 **Predicted vs measured axial wall static pressure distributions.**

with no probe installed at flow rates near that of Fig. 6. This distortion was caused by the probe support stem which was suspended from only one side of the meter. Its radial blockage varied with probe immersion and, although small, it produced a significant flowfield distortion which was large enough to produce the disagreement between predictions and measurements seen in Fig. 5. This data is key evidence that shows why static pressure survey devices should not be used in the throat of unchoked flow meters for discharge coefficient determination when high accuracies are required.

Measured Discharge Coefficients

Figure 8 shows the measured discharge coefficients based on the ASME critical flow metering nozzle calibrations compared with the pretest predictions for both natural transition and an all-turbulent boundary-layer assumption. The agreement between the predicted discharge coefficient for natural transition and the measured data is within $\pm 0.25\%$ everywhere except at the lowest pressure ratio of 1.01, where larger scatter was expected due to the known sensitivity of airflow to pressure measurement errors at low Mach number.[2] In order to investigate the influence of throat boundary-layer thickness due to transition location, a boundary-layer trip was located as shown in Fig. 9. Number 100 garnet sandblasting grit, having a mean particle diameter of 0.006 in., was used for the trip. This grit size was chosen to initiate transition immediately downstream of the trip using the criteria of Ref. 12. The trip location was chosen such that it was far enough upstream of the meter throat to represent as closely as possible an all-turbulent boundary layer, but far enough downstream such that relaminarization would not occur in the strong favorable pressure gradient in the contracting section. The difference in discharge coefficient between the tripped and untripped data is very close to the difference between the natural transition and all-turbulent boundary-layer predictions in Fig. 8.

Boundary-Layer Surveys

One throat boundary-layer survey was done with the trip installed, and three were done without a trip. Figure 10 shows comparisons of the measured profiles with the pretest predictions for a meter pressure ratio of 1.3027, and Fig. 11 compares the predicted displacement thickness with the measurements for the forced and natural transition cases. In general, the agreement is very good. For the natural transition cases, the difference between predicted and measured displacement thickness would represent an uncertainty in discharge coefficient of less than 0.03%.

Wall Static Pressure Distributions

Figures 12–14 show the axial distributions of measured wall static pressure compared with the predictions using the STC and Harris computer codes. The agreement is seen to be excellent over the full length of the meter for all flow rates (P_T/P_{SW}). The slight disagreement near the exit of the meter is characteristic of an uncoupled inviscid diffusing flowfield calculation which is adjusted for the calculated boundary-layer displacement thickness.

The excellent agreement between the measured wall pressure distributions and predictions, as well as between the predicted and measured boundary layers, provides added confidence that the agreement between the predicted and measured discharge coefficients in Fig. 8 is not just fortuitous.

Conclusions

A comprehensive subcritical flow meter design and test program has been conducted which clearly demonstrates that current theoretical methods can predict discharge coefficients to within 0.25% of those obtained from calibration with an industry standard critical flow ASME nozzle. To the author's knowledge, this is the only such program that has ever been conducted on a subcritical flow metering device which demonstrates this high level of accuracy.

Discharge coefficient measurements made with a boundary-layer trip, as well as meter throat boundary-layer surveys, clearly show the importance of transition location on accurate discharge coefficient predictions. Detailed surveys of the meter throat flowfield demonstrate that such surveys are not capable of measuring the radial static pressure profiles with sufficient accuracy to determine discharge coefficient to within 0.25% since the survey probe produces disturbances which alter the flowfield being measured. Freejet calibration of the survey probe is incapable of correcting for these effects since a freejet is not representative of the confined flowfield in the meter throat where probe blockage is important.

Comparisons of theoretically predicted wall static pressure distributions with measured data suggests that as long as the flow remains subsonic, the STC and Harris boundary-layer codes provide excellent predictions of the meter flowfield.

References

[1]Smith, R. E., and Matz, R. J., "A Theoretical Method of Determining the Discharge Coefficients for Venturis Operating at Critical Flow Conditions," *Transactions of the ASME, Series D, Journal of Basic Engineering*, Vol. 84, No. 4, 1962, pp. 435–446.

[2]Lahti, D. J., "Theory and Experiments on Subcritical Compressible Gas Flow Metering," Ph.D. Dissertation, Univ. of Cincinnati, Cincinnati, OH, Aug. 1990.

[3]Smith, R. E., and Matz, R. J., "Verification of a Theoretical Method of Determining Discharge Coefficients for Venturis Operating at Critical Flow Conditions," Arnold Engineering Development Center Rept. TR-61-8, Tullahoma, TN, Sept. 1961.

[4]Stratford, B. S., "The Calculation of the Discharge Coefficient of Profile Choked Nozzles and the Optimum Profile for Absolute Air Flow Measurement," *Journal of the Royal Aeronautical Society*, Vol. 68, April 1964, pp. 237–245.

[5]Anon., *Test Facilities Handbook*, 12th ed., Arnold Engineering Development Center, Arnold Air Force Station, TN, March 1984.

[6]Smith, R. E., and Matz, R. J., "Performance Characteristics of an 8 Inch Diameter ASME Nozzle Operating at Compressible and Incompressible Conditions," *Transactions of the ASME, Series J, Journal of Fluids Engineering*, Dec. 1973, pp. 542–550.

[7]Keith, J. S., Ferguson, D. R., Merkle, C. L., Heck, P. H., and Lahti, D. J., "Analytical Method for Predicting the Pressure Distribution About a Nacelle at Transonic Speeds," NASA CR-2217, July 1973.

[8]Harris, J. E., "Numerical Solution of the Equations for Compressible Laminar, Transitional, and Turbulent Boundary Layer Equations and Comparisons with Experimental Data," NASA TR R-368, Aug. 1971.

[9]Harris, J. E., and Blanchard, D. K., "Computer Program for Solving Laminar, Transitional, or Turbulent Compressible Boundary Layer Equations for Two-Dimensional and Axisymmetric Flow," NASA TM 83207, 1982.

[10]Anon., "Model Tests for Propulsion System Installed Performance," FluiDyne Engineering Corp. Brochure, Minneapolis, MN, Sept. 1990.

[11]Krause, L. N., "Effects of Pressure-Rake Design Parameters on Static-Pressure Measurement for Rakes Used in Subsonic Free Jets," NACA TN 2520, Oct. 1951.

[12]Braslow, A. L., Hicks, R. M., and Harris, R. V., "Use of Grit-Type Boundary Layer Transition Trips on Wind Tunnel Models," NASA TN-D3579, 1968.